

Massive Charged Scalar Quasinormal Modes of Reissner-Nördstrom Black Hole Surrounded by Quintessence

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Abstract. We evaluate the complex frequencies of the normal modes for the massive charged scalar field perturbations around a Reissner-Nördstrom black hole surrounded by a static and spherically symmetric quintessence using third order WKB approximation approach. Due to the presence of quintessence, quasinormal frequencies damp more slowly. We studied the variation of quasinormal frequencies with charge of the black hole, mass and charge of perturbing scalar field and the quintessential state parameter.

1. Introduction

It is now well understood that different kinds of perturbations in the geometry of a black hole can excite certain combination of its characteristic complex frequencies of the normal mode, called quasinormal modes(QNMs), whose real part represents the ring down frequency and imaginary part, the decay time and they are independent of the initial perturbation, but depends only on the parameters of black hole. The concept of QNM is put forward by Vishweshwara[2] and since then the quasinormal spectrum of black holes has been extensively studied for a great variety of black hole back grounds and perturbation fields because of its astrophysical and other theoretical interests. Astrophysical interests are associated with their relevance in gravitational wave analysis[3]. The QNMs of black hole are expected to be detected by the future gravitational wave detectors such as LISA[15], and give an opportunity to explore the properties of black holes directly. Apart from the observational interest of detection of quasinormal ringing by gravitational wave detectors, study of QNM found significance in AdS/CFT correspondence[17], black hole area quantization and Loop Quantum Gravity[16].

The idea that our universe is in a phase of accelerated expansion rather than holding steady, is strongly supported by a set of recent interlocking observations such as studies of distant supernovae[1], type 1a supernova[4] and cosmic microwave background radiation anisotropy[5], indicating the presence of some mysterious form of repulsive gravity called dark energy. In order to explain the nature of dark energy several models were proposed(for a recent review see[6]). The simplest option for this dark energy is Einstein's cosmological constant[7] with a constant equation of states(EoSs) $\epsilon = -1$ but it needs extreme fine tuning to account for the observations. The second is the dynamical scalar field models like quintessence[8], k-essence[9] and phantom[10], in which the EOSs of dark energy changes with time. Among them, the quintessence is the most natural model, and can give the EOSs with $-1 \leq \epsilon \leq -1/3$.

Because of relevance of the two topics, some studies of QNMs of black hole were started in the presence of quintessence, after Kiselev[18] derived the exact solution of Einstein equation for quintessential matter surrounding a black hole. QNMs of Schwarzschild black hole is studied for scalar[11], gravitational[12], electromagnetic[13] and massless Dirac field[14] perturbations in the presence of quintessence. Our work extend the studies to charged black hole space times. Charged scalar QNMs of Reissner-Nordstrom black hole were studied in[23, 24]. In this paper we considered the perturbation of Reissner-Nordstrom black hole encircled by quintessence for massive charged scalar field perturbations and study the dependence of quasinormal spectrum upon different parameters of the problem such as charge of the black hole, mass and charge of perturbing scalar field and the quintessential state parameter.

2. Massive charged scalar field around a Reissner-Nordstrom black hole surrounded by quintessence

Kiselev derived a static spherically symmetric exact solution of Einstein equations for quintessential matter surrounding a charged black hole under the condition of additivity and linearity in energy momentum tensor[18]. The metric can be expressed in the form

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2, \quad (1)$$

where

$$f(r) = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{c}{r^{3\epsilon+1}}\right) \quad \text{and} \quad d\Omega^2 = (d\theta^2 + \sin^2\theta d\phi^2),$$

with M and Q mass and charge of the black hole. ϵ is the quintessential state parameter and c is the normalization factor which depends on $\rho_q = \frac{-c}{2} \frac{3\epsilon}{r^{3(1-\epsilon)}}$, the density of quintessence. The Klein-Gordon equation describing the evolution of massive charged scalar perturbation field outside a charged black hole is given by[19]

$$\Phi_{;\mu\nu}g^{\mu\nu} - ieA_\mu g^{\mu\nu}(2\Phi_{;\nu} - ieA_\nu\Phi) - ieA_{\mu;\nu}g^{\mu\nu}\Phi = m^2\Phi, \quad (2)$$

where A_μ is the electromagnetic potential and e is the charge of the scalar field. Representing the charged scalar field in to spherical harmonics

$$\Phi = \frac{1}{r} \sum_{l,m} \eta_m^l(t, r) Y_l^m(\theta, \phi), \quad (3)$$

the wave equation becomes

$$\eta_{,tt} - 2ieA_t\eta_{,t} - \eta_{,r^*r^*} + f(r) \left[\frac{l(l+1)}{r^2} + \frac{f(r)'}{r} + m^2 \right] \eta - e^2 A_t^2 \eta = 0, \quad (4)$$

where we used the coordinate transformation defined by

$$dr^* = \frac{dr}{f(r)}. \quad (5)$$

The electromagnetic potential $A_t = C - \frac{Q}{r}$, where C is a constant. We define

$$\eta = e^{-ieCt} \Psi, \quad (6)$$

to avoid the physically unimportant quantity C . where Ψ is the radial part of perturbation variable taken to have time dependence $e^{-i\omega t}$. Now the radial perturbation equation can be written as

$$\Psi_{,r^*r^*} + \Theta\Psi = 0 \quad (7)$$

where $\Theta = \omega^2 - V^2$ and V is the scattering potential, which is a function of frequency of perturbation, ω and is given by

$$V = f(r) \left[\frac{l(l+1)}{r^2} + \frac{2M}{r^3} - \frac{2Q^2}{r^4} + \frac{c(3\epsilon+1)}{r^{3\epsilon+3}} + m^2 \right] + \frac{2eQ\omega}{r} - e^2 \frac{Q^2}{r^2} \quad (8)$$

3. Evaluation of Quasinormal modes

We use the third order WKB approximation method to determine the complex normal mode frequencies of black hole, a semi analytic method originally developed by Schutz and Will[20] for the lowest order. Later Iyer and Will[21] carried this approach to third WKB order and Konoplya[22] to sixth order to get more accurate results.

It gives a simple condition which will lead to discrete, complex values for the normal mode frequencies.

$$\frac{i\Theta_0}{\sqrt{2\Theta_0''}} - \Lambda(n) - \Omega(n) = n + \frac{1}{2}, \quad (9)$$

where Λ and Ω are higher order terms given by

$$\Lambda(n) = \frac{i}{(2\Theta_0'')^{1/2}} \left[\frac{1}{8} \left(\frac{\Theta_0^{(4)}}{\Theta_0''} \right) \left(\frac{1}{4} + \alpha^2 \right) - \frac{1}{288} \left(\frac{\Theta_0'''}{\Theta_0''} \right)^2 (7 + 60\alpha^2) \right], \quad (10)$$

$$\begin{aligned} \Omega = \frac{\alpha}{(2\Theta_0'')^{1/2}} & \left\{ \frac{5}{6912} \left(\frac{\Theta_0'''}{\Theta_0''} \right)^4 (77 + 188\alpha^2) - \frac{1}{384} \left(\frac{\Theta_0'''^2 \Theta_0^{(4)}}{\Theta_0''^3} \right) \right. \\ & (51 + 100\alpha^2) + \frac{1}{2304} \left(\frac{\Theta_0^{(4)}}{\Theta_0''} \right)^2 (67 + 68\alpha^2) + \frac{1}{288} \left(\frac{\Theta_0''' \Theta_0^{(5)}}{\Theta_0''^2} \right) \\ & \left. (19 + 28\alpha^2) + \frac{1}{288} \left(\frac{\Theta_0^{(6)}}{\Theta_0''} \right) (5 + 4\alpha^2) \right\}. \end{aligned} \quad (11)$$

where

$$\alpha = n + \frac{1}{2} \quad \text{with } n = \begin{cases} 0, 1, 2, \dots & \text{Re}(E) > 0 \\ -1, -2, -3, \dots & \text{Im}(E) < 0 \end{cases}$$

$\Theta_0^{(n)}$ denotes the n^{th} derivatives of Θ evaluated at r_0 , the value of r at which V attains maximum. Here a complexity arises from the fact that the potential V is a function of frequency ω . This makes difficult to calculate the value of r_0 . We make use of the procedure suggested by Konoplya[23] to find r_0 by fixing all the parameters other than ω for which the V depends and then find the value of r at which V attains maximum as a numerical function of ω . Substitute for r_0 and then find the value of ω which satisfies the condition (9) by trial and error way.

4. Results and Discussion

We take all the values of parameters in black hole mass(M) units. It is a general experience that the imaginary part of quasinormal frequencies decreases with increase in mode number, n , means that the quasinormal frequencies with lower mode number will decay slowly and are relevant to the description fields around black hole. So we consider frequencies for lower mode number for our study. The dependence of real and imaginary parts of ω on the charge of the black hole, Q is plotted in Figure1 for fixed

$l = 3, n = 0, e = 0.1, m = 0.1$ and for different values of ϵ with $c = 0.001$. The case with absence of quintessence is also plotted (dotted line).

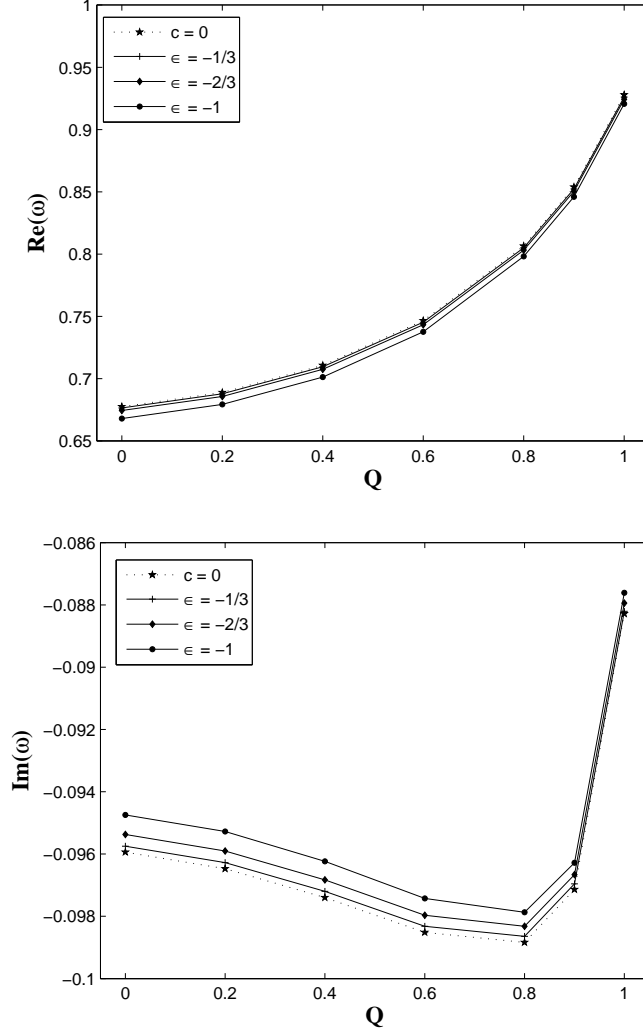


Figure 1. $\text{Re}\omega$ and $\text{Im}\omega$ as a function of Q for $l = 3, n = 0, e = 0.1, m = 0.1$ and for different values of ϵ with $c = 0.001$. The dotted line represents the no quintessence case($c=0$).

The quasinormal frequencies for scalar field in charged black hole is influenced by quintessence. The magnitudes of real and imaginary parts of ω is lower in the presence of quintessential field. This implies due to the presence of quintessence, the quasinormal frequencies for scalar field in RN black hole damps more slowly. As in the absence of quintessence[23, 24], $\text{Re}(\omega)$ increases monotonically with the increase in Q while the magnitude of $\text{Im}(\omega)$ first decreases, falling to a minimum around $Q = 0.8$ and thereafter increases sharply. For larger values of Q , the modification with quintessence decreases.

Figure2 shows the variation of $\text{Re}\omega$ and $\text{Im}\omega$ with quintessential state parameter ϵ with $c = 0.001$ for fixed $l = 3, n = 0, e = 0, m = 0.1$. As the value of ϵ increases $\text{Re}\omega$ increases slowly but the magnitude of $\text{Im}\omega$ increases more fast, means damping is less

for lower values of ϵ .

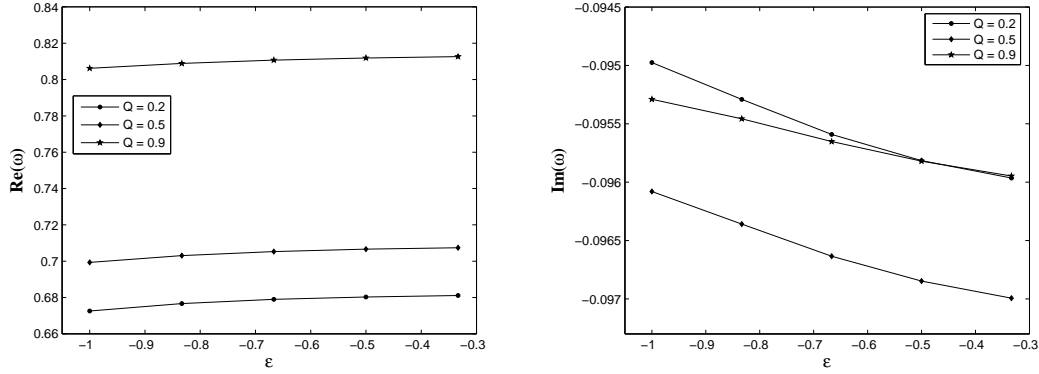


Figure 2. Variation of $\text{Re}\omega$ and $\text{Im}\omega$ with ϵ for $l = 3, n = 0, e = 0, m = 0.1$

In Figure3 $\text{Re}\omega$ and $\text{Im}\omega$ are plotted as functions of e with $l = 3, n = 0, m = 0.1$ for $Q = 0.1, 0.2, 0.3$ and different values of ϵ . Dotted line represents absence of quintessence ($c=0$). The variation is almost linear. In the presence of quintessence, the magnitude of $\text{Re}\omega$ and $\text{Im}\omega$ increases with e imitating the no quintessence case but with lower values of $\text{Re}\omega$ and $|\text{Im}\omega|$. The modification with e becomes larger for higher values of Q .

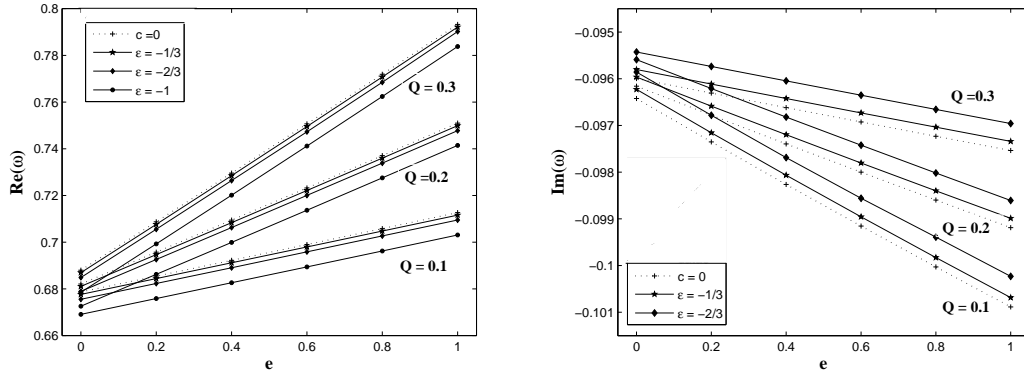


Figure 3. Variation of $\text{Re}\omega$ and $\text{Im}\omega$ with e for $l = 3, n = 0, m = 0.1, Q = 0.1, 0.2, 0.3$ and different values of ϵ . Dotted line is for $c = 0$.

Finally, we study the role of mass of scalar field on quasinormal frequencies. Quasinormal modes occur only when the peak value of the potential $V(r = r_{\text{max}})$ is larger than m^2 and ω^2 of the field is smaller than this peak value[25]. This means that there exists a maximum value for mass, m_{max} beyond which there will be no quasinormal modes. m_{max} can be estimated from the condition for the existence of quasinormal modes,

$$V(r_{\text{max}}, \omega = m_{\text{max}}) = (m_{\text{max}})^2 \quad (12)$$

The values of m_{max} obtained for different values of ϵ is tabulated in Table1. In the presence of quintessence m_{max} decreases because quintessence lowers the height of the potential barrier as shown in Figure5 and when $\epsilon = -1$, it has the lowest value.

c	ϵ	m_{max}	c	ϵ	m_{max}
0	—	0.88516	0.001	-2/3	0.87815
0.001	-1/3	0.88366	0.001	-1	0.85864

Table 1. The limit of mass of scalar field, m_{max} for the existence of quasinormal frequencies with $l = 3, e = 0, Q = 0.1$

The values of $Re\omega$ and $Im\omega$ evaluated using WKB method with the variation of mass is plotted in Figure4.

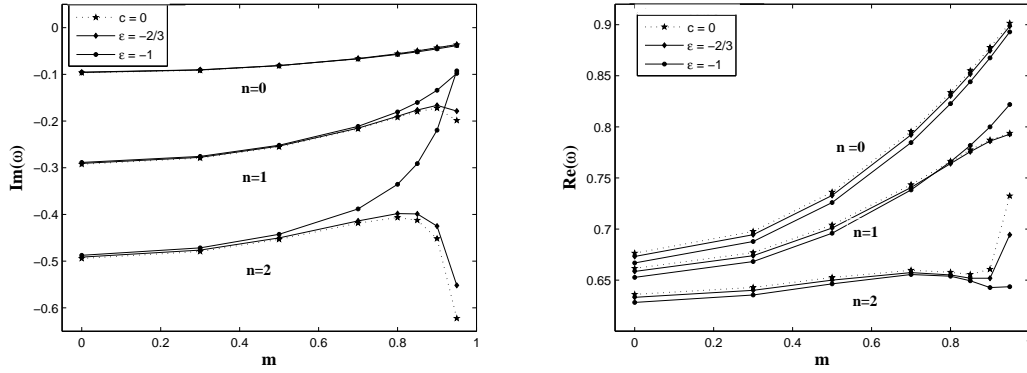


Figure 4. Variation of $Re\omega$ and $Im\omega$ with e for $l = 3, n = 0, m = 0.1, Q = 0.1, 0.2, 0.3$ and different values of ϵ . Dotted line is for $c = 0$.

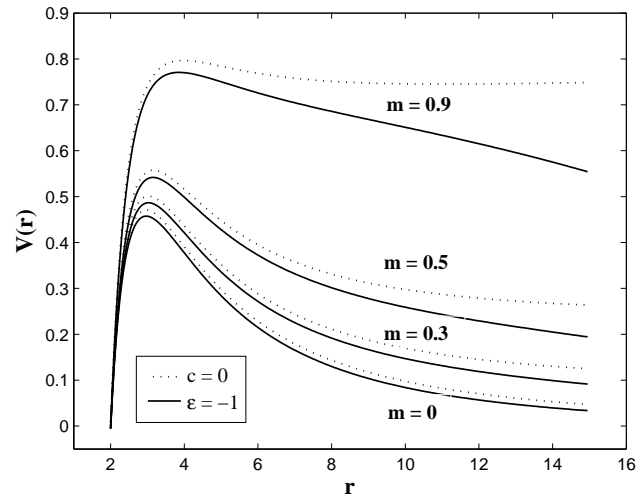


Figure 5. Effective potential $l = 3, n = 0, m = 0.1, Q = 0.1, 0.2, 0.3$ and different values of ϵ . Dotted line is for $c = 0$.

$Re\omega$ increases with increase in mass while $|Im\omega|$ decreases with mass, which indicates that QNMs of fields with lower mass damp slowly. But they behave abnormally near m_{max} . This is due to the fact that, for larger mass of the field the potential loses its barrier shape by broadening the potential peak as shown in Figure 5 and WKB method gives inaccurate results. Lower modes show less abnormality showing that WKB method is more accurate for fundamental modes.

Another thing we noticed is that this abnormal behavior is lower in the presence of quintessence and even when $\epsilon = -1$ we can get satisfactory curve because the peak of effective potential broadens much less in the presence of quintessence comparing with the normal case ($c=0$) as understood from Figure 5 and quintessence helps to retain barrier shape and give more accurate results at larger mass range.

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